Journal of Mechanical Science and Technology (KSME Int. J.), Vol. 20, No. 1, pp. 125～132, 2006

Stratified Steady and Unsteady Two-Phase Flows Between Two Parallel Plates

Woo-Gun Sim*
Hannam University,
133 Ojungdong Daedukgu, Taejon 306-791, Korea

To understand fluid dynamic forces acting on a structure subjected to two-phase flow, it is essential to get detailed information about the characteristics of two-phase flow. Stratified steady and unsteady two-phase flows between two parallel plates have been studied to investigate the general characteristics of the flow related to flow-induced vibration. Based on the spectral collocation method, a numerical approach has been developed for the unsteady two-phase flow. The method is validated by comparing numerical result to analytical one given for a simple harmonic two-phase flow. The flow parameters for the steady two-phase flow, such as void fraction and two-phase frictional multiplier, are evaluated. The dynamic characteristics of the unsteady two-phase flow, including the void fraction effect on the complex unsteady pressure, are illustrated.

Key Words : Spectral Collocation Method, Stratified Two-phase Flow, Void Fraction, Frictional Multiplier, Superficial Velocity, Slip Ratio, Unsteady Two-phase Flow

1. Introduction

Two-phase vapor-liquid flow exists in many shell and tube heat exchanger such as steam generators in Nuclear Steam Supply System. The elements of the heat exchanger can be subjected to excessive flow-induced vibrations, at a certain operating conditions, which can lead to fretting wear damage. Although fluid damping, fluidelastic stiffness and hydrodynamic mass in two-phase flow (Carlucci and Brown, 1983 ; Pettigrew et al., 2003) and in single-phase flow (Pettigrew and Taylor, 1991 ; Price, 1995) are reasonably well understood, little is known about the physical behavior of two-phase flow related to flow-induced vibration. Some knowledge of dynamic characteristics of two-phase flow is essential to formulate the problem related flow-induced vibration (Paidoussis et al., 1990). For reliable predictions of dynamics response of the elements, it is required to develop computer model for fluid damping and hydrodynamic mass. However, few numerical results (Hara and Kohgo, 1986) on hydrodynamic forces exist.

The homogeneous flow model and the separated flow model are the two most widely used and tested treatments of two-phase flow at present available. Schrage et al. (1988) took void fraction measurements in an in-line bundle with air-water cross-flow using quick-closing plate valves. They found that void fraction varies with mass flux and it is greatly over-predicted by the homogeneous equilibrium model. This model neglects the effect of the slip ratio based on the homogeneous model. For sufficient information about the characteristics of two-phase flow in horizontal tube bundles, an improved void fraction model has been developed by Feenstra et al. (2000). In a recent study, Pettigrew and Knowles (1997)

* **E-mail** : wgsim@hannam.ac.kr
TEL : +82-42-629-8089; **FAX** : +82-42-629-8293
Hannam University, 133 Ojungdong Daedukgu, Taejon 306-791, Korea. (Manuscript **Received** January 10, 2005; **Revised** December 12, 2005)

showed the effect of surface tension, using a chemical surfactant, on two-phase damping and on average bubbly size. However, in order to focus future studies on the flow-induced vibration, it is required to investigate the mechanism of the two-phase flow in detail, especially related to the damping and the added mass.

Recently, the spectral method (Mateescu et al., 1994) has been applied to the unsteady potential flow and then to the unsteady viscous flow in an eccentric annulus. The added mass and viscous damping were estimated, when a cylinder undergoes oscillatory motion in the plane of symmetry and normal to the plane of the symmetry. The viscous effects on the added mass and damping were evaluated, comparing the results obtained by potential flow theory with those obtained by the viscous flow theory. It was shown the viscous damping effect becomes important with decreasing annular space. To validate the spectral method, the results for potential flow were compared with the available analytical solutions of Chung and Chen (1977) for eccentric configurations and Fritz (1972) for concentric configurations. A study of free vibration of rectangular Mindlin plates was presented by Lee (2003), based on the Chebyshev pseudospectral method. The method uses test functions that satisfy the boundary conditions as basis functions. The result shows that rapid convergence and accuracy as well as the conceptual simplicity are achieved when the pseudospectral method is applied to the solution of eigenvalue problems. A FAMD (Fluid Added Mass and Damping) code was developed by Koo (2003) for practical applications calculating the fluid added mass and damping. In the formulations, a fluid domain is discretized with C0-type quadratic quadrilateral elements containing eight nodes using a mixed interpolation method.

Utilizing the spectral method (Mateescu et al., 1994), the stratified steady and unsteady two-phase flows between two parallel plates are evaluated. The effect of void fraction on the pressure, related to viscous damping and added mass, is illustrated for future study related to the flow-induced vibration.

2. Problem Formulations

2.1 Analytical solution of the two-phase flow

2.1.1 Steady flow

The two-phase steady flow is generated by the pressure drop along the axial direction as shown in Fig. 1(a). The plate is assumed to be infinitely long and the gap between the plates is H. Thus, for the parallel flow, we have $u=u(y)$, $v=w=\partial/\partial x=0$. As a result, the simplified Navier-Stokes equation for the steady flow is obtained,

$$\frac{\partial^2 u_f}{\partial y^2}=\frac{1}{\mu_f}\frac{\partial p_f}{\partial x}=-K_f \tag{1}$$

where the subscript f stands for both flows, gas and liquid, and K denotes $1/\mu(dp/dx)$. The solutions of steady flows (gas and liquid) are expressed as;

$$u_f=-\frac{1}{2}K_f y^2+C_{f1}y+C_{f2} \tag{2}$$

In the above equation, the four unknown coefficients C_{f1}, C_{f2} are determined by considering two fixed boundary conditions, $\vec{V}_f=0$, and two interface conditions ($u_g=u_l$ and $\tau_{yxg}=\tau_{yxl}$);

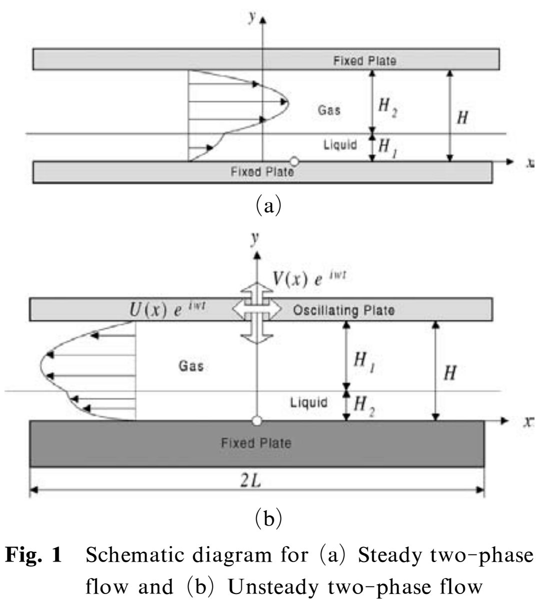


Fig. 1 Schematic diagram for (a) Steady two-phase flow and (b) Unsteady two-phase flow

$$
\begin{aligned}
&u_1=0 \text{ at } y=0, \quad u_2=0 \text{ at } y=H \\
&u_1=u_2, \quad \mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} \text{ at } y=H_1
\end{aligned} \tag{3}
$$

where μ represents viscosity of fluid. Thus, the solutions of the both flows can be expressed as ;

$$
\begin{aligned}
&u_1=-\frac{1}{2}K_1H_1^2\left[\left(\frac{y}{H_1}\right)^2-Ay\right] \\
&u_2=-\frac{1}{2}K_2H^2\left[\left(\frac{y}{H}\right)^2-1+A\frac{H_1^2}{H}(1-y/H)\right]
\end{aligned} \tag{4}
$$

where the constant, A, is

$$
A=\frac{1}{H+H_1(\mu_2/\mu_1-1)}\left[\frac{\mu_2}{\mu_1}+\left(\frac{H}{H_1}\right)^2-1\right]
$$

The mass flux of liquid or gas can be obtained by integrating the velocity across its height ;

$$
\begin{aligned}
&\dot{m}_1=-\frac{1}{6}\rho_1K_1H_1^3\left[1-\frac{3}{2}AH_1\right] \\
&\dot{m}_2=-\frac{1}{6}\rho_2K_2H^3[-2+3h+3AHh^2(1/2-h+h^2/2)-h^3]
\end{aligned} \tag{5}
$$

where the ratio of liquid height to total gap can be expressed as the void fraction, $h=H_1/H=\alpha$.

In order to investigate two-phase flow, it is convenient to define void fraction (α), superficial velocity (j), real flow velocity (u), flow mass quality (x) and two-phase frictional multiplier (ϕ), as follows ;

$$
\begin{aligned}
&\alpha=A_g/A, \quad j_g=Q_g/A, \; j_l=Q_l/A, \\
&u_g=Q_g/A_g, \; u_l=Q_l/A_l \\
&x=\dot{m}_g/(\dot{m}_g+\dot{m}_l), \; \left(\frac{dp}{dz}\right)_{two}=\left(\frac{dp}{dz}\right)_{single}\phi^2
\end{aligned} \tag{6}
$$

In the above equation, the subscripts, l and g, stand for liquid and gas, respectively. The two-phase frictional pressure gradient is expressed in terms of the single-phase pressure gradient for the total flow considered as liquid.

2.1.2 Unsteady flow by simple harmonic motion

The unsteady two-phase flow is given by simple harmonic motion, $u=U_0e^{i\omega t}$ of upper plate, while the lower plate is fixed, as shown in Fig. 1(b). Based on small amplitude motion, the momentum equation can be simplified as ;

$$
4i\mathrm{Re}_{sf}\hat{u}_f=\frac{\partial^2\hat{u}_f}{\partial\hat{y}^2} \tag{7}
$$

where $i=\sqrt{-1}$, $\hat{u}_f=u_f/U_oe^{i\omega t}$, $\hat{y}=y/H$ and $\mathrm{Re}_{sf}=\rho_f\omega H^2/4\mu_f$. The analytical solution for $\hat{u}_f$ may be written in the form

$$
\hat{u}_f=A_{f1}e^{\alpha_f\hat{y}}+A_{f2}e^{-\alpha_f\hat{y}} \tag{8}
$$

where the complex constant α_f is expressed as $\alpha_f=(1+i)\sqrt{\mathrm{Re}_{sf}/2}$. The a priori unknown constants, A_{f1} and A_{f2} are determined by considering two boundary conditions and two interface conditions between two fluids ;

$$
\begin{aligned}
&\hat{u}_1=1 \text{ at } \hat{y}=1, \quad \hat{u}_2=0 \text{ at } \hat{y}=0 \\
&\hat{u}_1=\hat{u}_2, \quad \frac{\partial\hat{u}_1}{\partial\hat{y}}=\frac{\mathrm{Re}_{s1}}{\mathrm{Re}_{s2}}\frac{\rho_2}{\rho_1}\frac{\partial\hat{u}_2}{\partial\hat{y}} \text{ at } \hat{y}=\hat{y}_2
\end{aligned} \tag{9}
$$

Considering the boundary and interface conditions, we can get the system equation in matrix form ;

$$
\begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ 0 & 0 & M_{43} & M_{44} \end{bmatrix}
\begin{Bmatrix} A_{11} \\ A_{12} \\ A_{21} \\ A_{22} \end{Bmatrix}
=\begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}
$$

where

$M_{11}=e^{\alpha_1}$, $M_{12}=e^{-\alpha_1}$, $M_{21}=e^{\alpha_1\hat{y}_2}$,
$M_{22}=e^{-\alpha_1\hat{y}_2}$, $M_{23}=-e^{\alpha_2\hat{y}_2}$, $M_{24}=-e^{-\alpha_2\hat{y}_2}$,
$M_{31}=\alpha_1e^{\alpha_1\hat{y}_2}$, $M_{32}=-\alpha_1e^{-\alpha_1\hat{y}_2}$,
$M_{33}=-\frac{\mathrm{Re}_{s1}}{\mathrm{Re}_{s2}}\alpha_2e^{\alpha_2\hat{y}_2}$, $M_{34}=\frac{\mathrm{Re}_{s1}}{\mathrm{Re}_{s2}}\alpha_2e^{-\alpha_2\hat{y}_2}$,
$M_{43}=1$, $M_{44}=1$

from which the constants are obtained.

2.2 Numerical approach for two-phase flow

2.2.1 Spectral collocation method

A numerical approach for the stratified two-phase flow has been developed based on the spectral collocation method. By assuming a small amplitude motion of the moving body for unsteady flow, linear governing equations are obtained. As a result, the governing equation of the flows, represented by the Navier-Stokes and continuity equations, may be expressed in matrix form as

$$E\left(\frac{\partial \vec{v}}{\partial t}, \frac{\partial \vec{v}}{\partial x}, \frac{\partial \vec{v}}{\partial y}, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial^2 \vec{v}}{\partial x^2}, \frac{\partial^2 \vec{v}}{\partial y^2}, \rho, \mu\right)=0 \quad (11)$$

where t and x, y are independent variables representing the time (for unsteady flow) and geometrical coordinates, respectively. Usually, the variables and parameters appearing in Eq. (11) are conveniently expressed in dimensionless form, instead of the actual physical parameters, e.g.; $\theta=\pi x/2L$ and $\hat{y}=2y/H-1$. The above equations are subjected to specific boundary conditions, moving and fixed boundary conditions.

Based on no slip condition between fluid and plates, the boundary conditions can be written as

$$\vec{V}_f=\vec{V}_B \quad (12)$$

where subscripts f and B stand for fluid and body, respectively. To solve the present stratified flows, it is required to consider interface conditions where flow parameters (pressure and velocity vectors) and axial skin friction of both gas and liquid flows are the same;

$$p_g=p_l,\ \vec{V}_g=\vec{V}_l,\ \mu_g\frac{\partial u_g}{\partial y}=\mu_l\frac{\partial u_l}{\partial y} \quad (13)$$

where subscripts, g and l, represent gas and liquid, respectively.

The present spectral method (Sim and Kim, 1996) is based on suitable spatial expansion for the dimensionless fluid dynamic parameters of the unsteady flow. The flow parameters can be expressed in terms of Chebyshev polynomials, T_j, and Fourier functions, F, such as $\sin(\pi x/2L)$ or $\cos(\pi x/2L)$;

$$\hat{U}_f(\theta, \hat{y})=\sum_{j=0}^{m} U_{fj}T_j(\hat{y})F(\theta)e^{i\omega t} \quad (14)$$

where ω denotes the oscillatory frequency and U_{fj} are a priori unknown complex coefficients. The spatial expansion is usually performed in the computational domain, obtained by a convenient transformation from the physical domain.

To get the system equation with this spectral expansion, the governing equations are imposed at specified collocation points within computational domain. Using the collocation approach, algebraic systems of equations can be obtained from Eqs. (11) ~ (14);

$$[A]\{U_{fj}\}=\{b\} \quad (15)$$

where b is related to moving boundary conditions. This system of equations in complex form is solved for the unknown complex coefficients of the spectral expansions of the fluid parameters.

2.2.2 Unsteady two-phase flow

As shown in Fig. 2(b), the present numerical approach is applied to two-dimensional two-phase flow generated by more complicated moving boundary conditions; $u_B=U_o\sin(\pi x/2L)e^{i\omega t}$ and $v_B=V_o\cos(\pi x/2L)e^{i\omega t}$. The product terms between the unsteady components are neglected by the assumption of small amplitude motion of the plate. (e.g.; $u\cdot\partial v/\partial x\ll 1$). Thus, for the unsteady viscous incompressible flow, continuity equation and Navier-Stokes equations can be expressed in linear form;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (16)$$
$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

subjected to boundary and interface conditions;

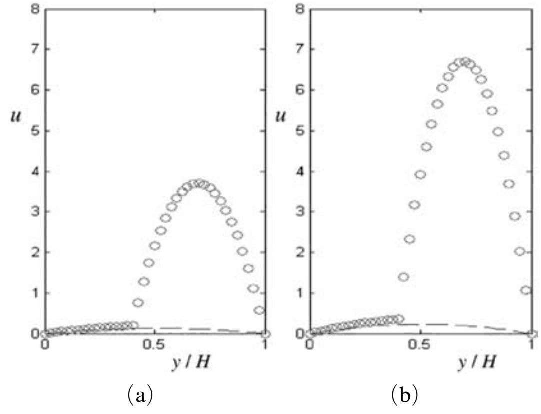


Fig. 2 Velocity profile of steady flow for $H_l/H=0.4$, $\mu_l/\mu_g=80$, $\rho_l/\rho_g=1500$ and $-1/\mu_l(dp/dx)=1$ with (a) $dp_{tp}/dx=dp_{sp}/dx$ and (b) $\dot{m}_g+\dot{m}_l=\dot{m}_s$ (○; Two-phase flow, ---; Single-phase liquid flow)

$$
\begin{aligned}
&v=0,\ u=0 && \text{at } y=0\\
&u_1=U_o\sin(\pi x/2L)\,e^{i\omega t} && \text{at } y=H\\
&v_1=V_o\cos(\pi x/2L)\,e^{i\omega t} && \text{at } y=H\\
&v_1=v_2,\ u_1=u_2,\ p_1=p_2 && \text{at } y=H_2\\
&\mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} && \text{at } y=H_2
\end{aligned}
\qquad (17)
$$

In order to generalize the present problem, it is convenient to define the following dimensionless parameters ;

$$
\begin{aligned}
&\hat{v}=\frac{v}{V_o e^{i\omega t}},\ \hat{u}=\frac{u}{U_o e^{i\omega t}},\ \hat{p}=\frac{p}{i\rho\omega H V_o e^{i\omega t}}\\
&\mathrm{Re}_{sf}=\frac{\rho_f\omega H^2}{4\mu_f},\ \theta=\frac{x}{2L}\pi,\ \hat{y}=\frac{2y}{H}-1\\
&\hat{l}=\frac{2L}{\pi H},\ h=\frac{H_2}{H_1}
\end{aligned}
\qquad (18)
$$

where Re_{sf} denotes the oscillatory Reynolds number of fluid f. The flow-dynamic properties are expressed in expansion forms. Considering the geometry, the flow parameters are expressed as

$$
\begin{aligned}
&\hat{v}(\theta,\hat{y})=\sum_{k=0}^{n}\sum_{j=0}^{m}V_{jk}T_j(\hat{y})\,c(k\theta)\\
&\hat{u}(\theta,\hat{y})=\sum_{k=0}^{n}\sum_{j=0}^{m}U_{jk}T_j(\hat{y})\,s(k\theta)\\
&\hat{p}(\theta,\hat{y})=\sum_{k=0}^{n}\sum_{j=0}^{m}P_{jk}T_j(\hat{y})\,c(k\theta)
\end{aligned}
\qquad (19)
$$

where $c(k\theta)=\cos(k\theta)$, $s(k\theta)=\sin(k\theta)$. The choice of Fourier series as an interpolation function in the axial direction stems from of the periodic character of the flow.

Considering the expansion forms, the governing equations can be expanded as

$$
\sum_{k=0}^{n}\sum_{j=0}^{m}\left(U_{jk}kT_j(\hat{y})\,c(k\theta)+2\hat{l}\,V_{jk}T_j'(\hat{y})\,c(k\theta)\right)=0
$$

$$
\sum_{k=0}^{n}\sum_{j=0}^{m}\left[\begin{array}{l}U_{jk}\left(\begin{array}{c}\left(i\mathrm{Re}_{sf}+\dfrac{k^2}{4\hat{l}^2}\right)T_j(\hat{y})\,s(k\theta)\\ -T_j''(\hat{y})\,s(k\theta)\end{array}\right)\\ -P_{jk}\dfrac{ik\mathrm{Re}_{sf}}{\hat{l}}\,T_j(\hat{y})\,s(k\theta)\end{array}\right]=0 \qquad (20)
$$

$$
\sum_{k=0}^{n}\sum_{j=0}^{m}\left[\begin{array}{l}V_{jk}\left(\begin{array}{c}\left(i\mathrm{Re}_{sf}+\dfrac{k^2}{4\hat{l}^2}\right)T_j(\hat{y})\,s(k\theta)\\ -T_j''(\hat{y})\,c(k\theta)\end{array}\right)\\ +P_{jk}i2\mathrm{Re}_{sf}T_j'(\hat{y})\,c(k\theta)\end{array}\right]=0
$$

subjected to boundary conditions ;

$$
\begin{aligned}
&\sum_{k=0}^{n}\sum_{j=0}^{m}V_{jk}T_j(1)\,c(k\theta)=1\\
&\sum_{k=0}^{n}\sum_{j=0}^{m}U_{jk}T_j(1)\,s(k\theta)=1\\
&\sum_{k=0}^{n}\sum_{j=0}^{m}V_{jk}T_j(-1)\,c(k\theta)=0\\
&\sum_{k=0}^{n}\sum_{j=0}^{m}U_{jk}T_j(-1)\,s(k\theta)=0
\end{aligned}
$$

The formulae for the derivatives of Chebyshev polynomials are

$$
L(f(z))=\sum_n b_m T_n(z); \qquad (21)
$$

$$
f'(z);\ b_n-\frac{2}{c_n}\sum_{p=n+1}pa_p, \qquad n+p=odd
$$

$$
f''(z);\ b_n=\frac{1}{c_n}\sum_{p=n+2}p(p^2-n^2)\,a_p,\ n+p=even
$$

Using the collocation approach mentioned in section 2.2.1, algebraic systems of equations can be obtained from Eqs. (20) and (21); $[A]\{V\}=\{B\}$, where the column vector V stands for a prior unknown coefficients and the column vector B is related to the boundary conditions. In the determination of the unknown coefficients of the spectral expansion, it is necessary to assign more collocation points in the computational domains. The present numerical solution is obtained with $n=m=6$ collocation points which are uniformly distributed axially and vertically.

3. Typical Results and Discussions

3.1 Steady two-phase flow

For the purpose of understanding two-phase flow mechanisms and for the future purpose of estimation of steady viscous forces, the analysis for steady two-phase flow has been performed. The velocity profile is shown in Fig. 2 for $H_1/H=0.4$, $\mu_l/\mu_g=80$, $\rho_l/\rho_g=1500$ and $-1/\mu_l(dp/dx)=1$. The result in Fig. 2(a) is calculated for $dp_{tp}/dx=dp_{sp}/dx$ while in (b) for $\dot{m}_g+\dot{m}_l=\dot{m}_s$. The dotted line is given for single-phase liquid. It is found that (1) the velocity in gas flow is higher than that in liquid flow, (2) the mass flow rate of two-phase flow for a given pressure drop per unit length, is lower than that of single

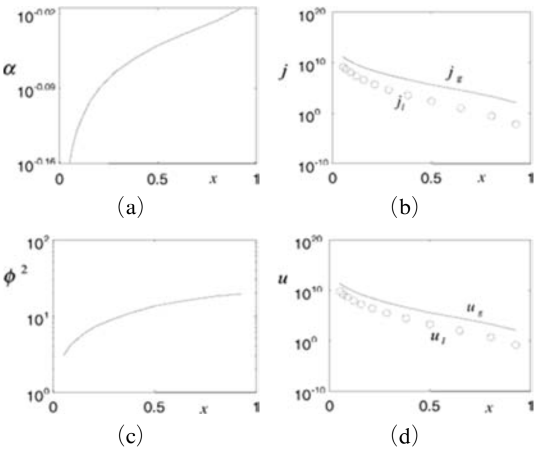


Fig. 3 Fluid parameters for two-phase flow versus quality; (a) Void fraction, (b) Volumetric flux, (c) Two-phase frictional multiplier and (d) Real flow velocity

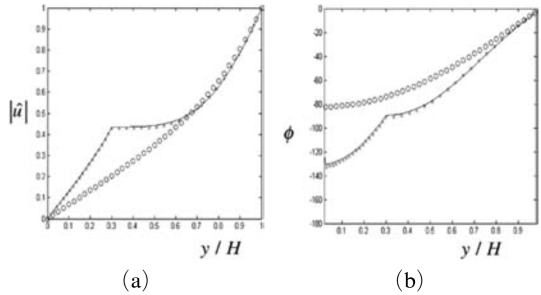


Fig. 4 Comparison of typical numerical results (∗), to Semi-analytical Results (−) for simple harmonic two-phase flow ($Re_{s1}=2.5$, $Re_{s2}=12.5$, $\rho_2/\rho_1=0.01$, $\hat{l}=\infty$ and $\alpha=0.3$): (a) Amplitude and (b) Phase angle of dimensionless unsteady flow (○; Single phase liquid flow)

phase liquid flow; $\dot{m}_g+\dot{m}_l<\dot{m}_s$ and (3) the pressure drop of the two-phase flow is higher than that of single phase flow for a given mass flow rate. The ratio of these pressure drops is defined by two-phase frictional multiplier, $\phi^2=(dp_{tp}/\mathrm{d}x)/(dp_{sp}/dx)$ which is shown in Fig. 3 (c). The other important fluid parameters of two-phase flow, (a) void fraction ($\alpha=A_g/A$), (b) superficial velocity ($j_g=Q_g/A$, $j_l=Q_l/A$) and (d) real flow velocity ($u_g=Q_g/A_g$, $u_l=Q_l/A_l$) versus flow quality ($x=\dot{m}_g/(\dot{m}_g+\dot{m}_l)$), are presented in Fig. 3. It is shown that the volumetric flux and real flow velocity of gas phase are higher than those of liquid phase.

3.2 Unsteady two-phase flow

To validate the present numerical approach for two-phase flow, the problem is formulated for simple harmonic two-phase flow (one dimensional unsteady flow), generated by harmonic oscillatory motion of the moving plate at $y=H$ in the tangential direction to the plate; $u=U_o e^{i\omega t}$. The dimensionless amplitude of unsteady velocity, $|\hat{u}|=|u/(U_o e^{i\omega t})|$, and phase angle, $\phi=\tan^{-1}\{\mathrm{Im}(\hat{u})/\mathrm{Re}(\hat{u})\}$, are given in Fig. 4 by the numerical approach for, $Re_{s1}=\rho_1\omega H^2/4\mu_1=2.5$, $Re_{s2}=\rho_2\omega H^2/4\mu_2=12.5$, $\rho_2/\rho_1=0.01$, $\hat{l}=2L/\pi H=\infty$ and $\alpha=H_2/H=0.3$. The numerical results are compared to the semi-analytical results given by Eq. (10). Good agreement is found between the numerical results and the analytical one given for the two-phase flow. As compared to the results for single-phase liquid flow (o), liquid flow ($y/H>0.3$) is less restricted by the fixed plate ($y/H=0$). So, the amplitude of velocity of two-phase flow is higher than that of single-phase flow.

The present numerical results for two-dimensional two-phase flow ($Re_{s1}=1$, $Re_{s2}=1$, $\rho_1=0.01$, $\hat{l}=4$ and $\alpha=0.2$) are illustrated in Fig. 5. To see the two-phase effect on the flow parameters, the result of single-phase liquid is denoted by a dotted line in Fig. 5. Similarly to the result of steady flow shown in Fig. 2, the axial component, $\hat{u}$, of gas phase flow is higher than that of the liquid phase; however, vertical components, $\hat{v}$, is less than 1 in amplitude. It is found that the amplitude of unsteady pressure for two-phase flow is less than those of single-phase flow and there is no pressure variation in the vertical direction. At a certain time ($e^{i\omega t}\approx 1$), the velocity vectors and pressure of unsteady flow are shown in Fig. 6 for the same flow parameters given for Fig. 5 except $\alpha=0.3$. It is clear that the first order term of Fourier series defined in Eq. (19) is dominant and the vertical variation of the dimensionless axial velocity is similar to the results given for the steady flow (see Fig. 2). The void

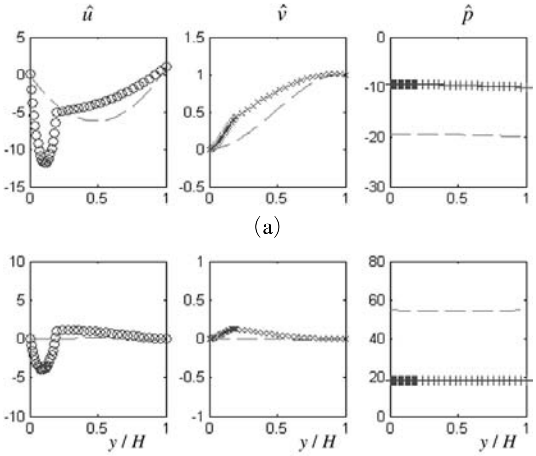


Fig. 5 Typical results for two-dimensional two-phase flow for $Re_{s1}=1$, $Re_{s2}=1$, $\rho_2/\rho_1=0.01$, $\hat{l}=4$ and $\alpha=0.2$; (a) Real and (b) Imaginary components of dimensionless unsteady flow parameter (---; Single phase liquid flow)

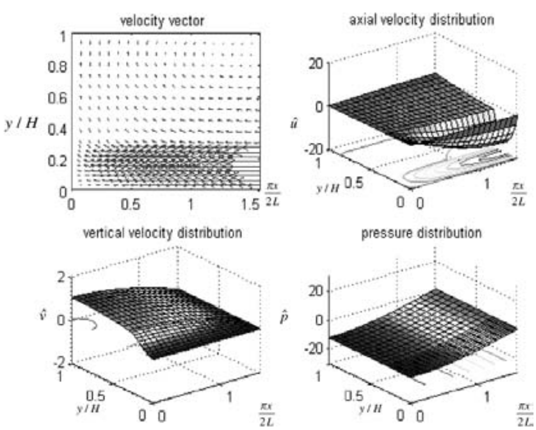
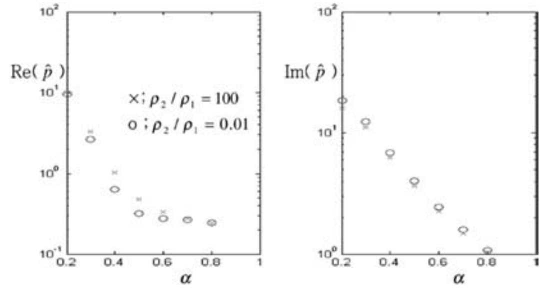


Fig. 6 Velocity vectors and pressure of unsteady flow in a certain time $(e^{i\omega t}\approx 1)$ ($Re_{s1}=1$, $Re_{s2}=1$, $\rho_2/\rho_1=0.01$, $\hat{l}=4$ and $\alpha=0.3$)

fraction effect on the complex pressure for $Re_{s1}=1$, $Re_{s2}=1$, $\hat{l}=4$ and $\rho_2/\rho_1=0.01$ (o) or 100 ($\times$), is illustrated in Fig. 7. Since, the complex pressure is defined with respect to acceleration, the real part and imaginary parts of the pressure component are related to added mass and viscous damping, respectively. Thus, it is expected that added mass and viscous damping decrease with void fraction. It is found the dimensionless pressure is less affected by the oscillatory Reynolds numbers, Re_{s1} and Re_{s2}.



(a) (b)

Fig. 7 Dimensionless amplitude of unsteady pressure versus void fraction; (a) Real and (b) Imaginary components ($Re_{s1}=1$, $Re_{s2}=1$ and $\hat{l}=4$)

5. Conclusions

A numerical approach based on the spectral collocation method has been developed for stratified two-phase unsteady flow between two parallel plates. It is essential to investigate the dynamic characteristics of the two-phase flow, for the future studies related to the flow-induced vibration. Analytical solutions for a steady two-phase flow and simple harmonic two-phase flow are provided to verify the present numerical method. The numerical method is based on suitable spatial expansions of fluid-dynamic parameters, using Chebyshev polynomials and Fouier functions. The unknown complex coefficients contained in these expansions are determined by applying the collocation method to the governing equations, boundary conditions and interface conditions. This method is validated by comparing numerical results to analytical result for simple harmonic two-phase flow. General dynamic characteristics of two-phase flow are shown from the results of the steady two-phase flow; e.g., two-phase frictional multiplier is higher than 1 and real flow velocity of gas flow is higher than that of liquid flow. It is found that the dimensionless pressure decreases with void fraction for the stratified unsteady two-phase flow. The effect of the oscillatory Reynolds number on the dimensionless pressure is minor as compared to the effect of the void fraction.

References

Carlucci, L. N. and Brown, J. D., 1983, "Experimental Studies of Damping and Hydrodynamic Mass of a Cylinder in Confined Two-phase Flow," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 105, pp. 83~89.

Chung, H. and Chen, S. S., 1977, "Vibration of a Group of Circular Cylinders in a Confined Fluid," *Journal of Applied Mechanics*, Vol. 44, pp. 213~217.

Feenstra, P. A., Weaver, D. S. and Judd, R. L., 2000, "An Improved Void Fraction Model for Two-Phase Cross-Flow in Horizontal Tube Bundles," *International Journal of Multiphase Flow*, Vol. 26, pp. 1851~1873.

Fritz, R. J., 1972, "The Effect of Liquids on the Dynamic Motions of Immersed Solids," *ASME Journal of Engineering for Industry*, Vol. 94, pp. 167~173.

Hara, F. and Kohgo, O., 1986, "Numerical Approach to Added Mass and Damping of a Vibrating Circular Cylinder in a Two-phase Bubble Fluid," *Proc. ICCM, Tokyo*, Vol. 2, pp. 255~260.

Koo, G.-H. and Lee, J.-H., 2003, "Development of FAMD Code to Calculate the Fluid Added Mass and Damping of Arbitrary Structures Submerged in Confined Viscous Fluid," *KSME International Journal*, Vol. 17, No. 3, pp. 457~466.

Lee, J. H., 2003, "Eigenvalue Analysis of Rectangular Mindlin Plates by Chebyshev Pseudospectral Method," *KSME International Journal*, Vol. 17, No. 3, pp. 370~379.

Mateescu, D., Paidoussis, M. P. and Sim, W. G., 1994, "Spectral Solutions for Unsteady Annular Flows between Eccentric cylinders Induced by Transverse Oscillations," *Journal of Sound and Vibration*, Vol. 177, No. 5, pp. 635~649.

Paidoussis, M. P., Mateescu, D. and Sim, W. G., 1990, "Dynamics and Stability of a Flexible Cylinder in a Narrow Coaxial Cylindrical Duct Subjected to Annular Flow," *Journal of Applied Mechanics*, Vol. 57, pp. 232~240.

Pettigrew, M. J. and Taylor, C. E., 1991, "Fluidelastic Instability of Heat Exchanger Tube Bundles : Review and Design Recommendations," *ASME Journal of Pressure Vessel Technology*, Vol. 113, pp. 242~256.

Pettigrew, M. J. and Taylor, C. E., 2003, "Vibration Analysis of Shell-and-tube Heat Exchangers ; an Overview- Part 1 : Flow, Damping, Fluidelastic Instability," *Journal of Fluids and Structure*, Vol. 18, pp. 469~483.

Pettigrew, M. J. and Knowles, G. D., 1997, "Some Aspects of Heat-Exchanger Tube Damping in Two-Phase Mixtures," *Journal of Fluids and Structure*, Vol. 11, pp. 929~945.

Price, S. J., 1995, "A Review of Ttheoretical Models for Fluidelastic Instability of Cylinder Arrays in Cross-flow," *Journal of Fluids and Structure*, Vol. 9, pp. 463~518.

Schrage, D. S., Hsu, J. T. and Jensen, M. K., 1988, "Two-phase Pressure Drop in Vertical Cross Flow across a Horizontal Tube Bundle," *AIChE J.*, Vol. 34, pp. 107~115.

Sim, W. G. and Kim, J. M., 1996, "Application of Spectral Collocation Method to Conduction and Laminar Forced Heat Convection in Eccentric Annuli," *KSME Journal*, Vol. 10, No. 1, pp. 125~132.